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CHROMATIC VARIATION OF SPHERICAL ABERRATION IN CEMENTED DOUBLETS

BY MISS L. M. GILLMAN, B.Sc.

(From the National Physical Laboratory)

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THE determination of the form of achromatic doublets, designed to give freedom from certain aberrations, has been the subject of numerous investigations, the results of which have been of great practical importance. Reference may be made to the recent valuable addition to English optical literature in the translation of Steinheil and Voit's *Applied Optics* by J. W. French and to Harting's research on double and triple cemented objectives free from spherical aberration and coma. An account of Harting's work, together with other investigations of the same nature, is to be found in Gleichen's *Lehrbuch der geometrischen Optik* and, for the use of the British manufacturer in particular, tables* containing data for the forms of thin telescope objectives have been calculated at the National Physical Laboratory. With the exception of the work of Steinheil and Voit, which was carried out by trigonometrical computation, the method employed in the works referred to has been algebraical. The results obtained in this way are necessarily approximate and it is desirable that they should be tested by applying the rigorous methods of ray tracing to determine the aberrations still remaining. In order to discover the possibility of improvement on the calculated system, not only should the objective in question be examined in this way but also similar objectives differing from it by slight variations in its surfaces. This is the procedure adopted in the investigation of which the present paper is an account. Advantage has been taken of a proposed Standard Tool List published in the last volume of the *Transactions*

* "Constructional Data of Small Telescope Objectives," and "Additional Data for the Construction of Small Telescope Objectives," by T. Smith and R. W. Cheshire.

*of the Optical Society** which has been used to determine the steps by which the curvature of the surfaces should be varied. The results of these calculations will thus show, additionally, what influence the proposed spacing of the tools has on the aberrations of the lens system under consideration.

Types of cemented doublets have been designed with either the flint or the crown lens leading. The latter may be divided into two distinct classes, depending on the refractivity of the crown glass employed. With an ordinary silicate crown glass of low refractive index, the crown lens is double convex with surfaces of very nearly equal curvatures, and is cemented to a flint lens which is practically plano-concave. Such a combination when corrected for central spherical aberration is troubled with considerable coma. The second class includes objectives made of a more highly refracting crown, a medium barium crown being generally used. In this case, the double convex crown lens has unequal curvatures, the outside surface being much shallower than the cemented surface, while the outside flint surface has the least curvature of any. This class of objective has the advantage that with the correction of spherical aberration, there is much less coma than when a crown lens of low refractive index is used. Sets of objectives belonging to each of these three types have been examined in the following investigation and they will be hereafter referred to, according to their type, as Set I (ordinary silicate crown leading), Set II (flint leading followed by ordinary silicate crown), Set III (medium barium crown leading), each set consisting of slightly varying forms of one member of each type.

The surfaces of the original system from which a set was derived were determined by algebraical methods, without reckoning thicknesses, to give a lens of focal length 250 mm. (10″), adjusted to give coincidence for the foci for *C* and *F* rays and corrected for first order spherical aberration for *D*. The nearest tool numbers to these calculated curvatures were selected from the Standard Tool List referred to. Each set was constructed by making each original tool number the centre of a set of consecutive tool numbers, so obtaining series of regularly varying surfaces, which were then combined in all possible ways. In the three sets so obtained, the first two surfaces were given five different curvatures, but in Set II only was the third surface varied and in this case only two curvatures were used giving Sets II A and B. A change in the third surface affects the spherical aberration very little, so that a more complete investigation would be unprofitable. In the following tables are collected structural details for each set, including

(*a*) the optical properties of the glasses used, taken from Messrs Chance Bros.' list;

(*b*) the calculated values for the curvatures of the typical lens;

(*c*) the tool numbers of the list referred to used successively for the different surfaces;

(*d*) the curvatures of the tools in dioptres.

* "The Spacing of Glass Working Tools," by T. Smith, *Trans. Opt. Soc.* 20 (1919), 237.

Constructional Data of Sets

Set I. Crown lens leading:

Glasses used		Refractive indices C	D	F	G′
No. 9322	Hard crown	1·51606	1·5186	1·52466	1·52952
360	Dense flint	1·61758	1·6225	1·63487	1·64539

Curvatures of the calculated doublet in dioptres: 9·520, 9·318, 0·051.

Tool numbers for 1st surface Nos. 136–140
" " 2nd " Nos. 137–141
" " 3rd " No. 0 (flat)

Curvatures of tools in dioptres:

No. 136	9·221	No. 139	9·496
137	9·312	140	9·588
138	9·404	141	9·680

Set II. Flint lens leading:

Glasses used—same as in Set I.

Curvatures of the calculated doublet in dioptres: 5·918, 15·289, 3·555.

Tool numbers for 1st surface Nos. 95–99
" " 2nd " Nos. 193–197
" " 3rd " No. 63 (Set II A) and No. 64 (Set II B)

Curvatures of tools in dioptres:

No. 95	5·798	No. 193	15·035	No. 63	3·524
96	5·874	194	15·148	64	3·590
97	5·951	195	15·263		
98	6·028	196	15·378		
99	6·106	197	15·493		

Set III. Crown lens leading:

Glasses used		Refractive indices C	D	F	G′
No. 9002	Medium barium crown	1·57148	1·5744	1·58143	1·58711
370	Dense flint	1·60708	1·6118	1·62365	1·63370

Curvatures of the calculated doublet in dioptres: 6·888, 12·126, 0·710.

Tool numbers for 1st surface Nos. 106–110
" " 2nd " Nos. 164–168
" " 3rd " No. 16

Curvatures of tools in dioptres:

No. 106	6·658	No. 164	11·914	No. 16	0·777
107	6·739	165	12·016		
108	6·819	166	12·118		
109	6·900	167	12·221		
110	6·982	168	12·324		

In the calculations the values for the curvatures and refractive indices of the glasses used have been those already given in the tables; the axial thicknesses of the lenses were estimated for an aperture of 38 mm. ($1\frac{1}{2}$"), by allowing, in the case of convex lenses, a marginal thickness of 1 mm. to be added to the thickness due to curvature and, in the case of concave lenses, an axial thickness of 1 mm., so that the axial thickness of the crown lens varied from 4·71 to 4·86 mm., while that of the flint lens was always 1 mm. Both axial and marginal rays, incident parallel to the optical axis, were traced through the lens for colours *C*, *D*, *F*, and *G'*. The errors are exhibited by drawing separate curves showing, for each lens, the variation in the position of the intersection points for paraxial and marginal rays of all these colours, distances from the third surface being measured along the abscissae, wave-lengths along the ordinates. It was found convenient to group together curves for the same first surface with varying values of the second surface.

Results for Sets I, II A and B, and III are shown in Figs. 1, 2 A and B, and 3. Paraxial results are given in full curves, marginal results in dotted curves. The common tool number for the first surface is placed at the side of each group; the tool number for the second surface is placed over each pair of curves.

The variation in focal length of the lenses, as the surfaces change, is clearly seen, as is also the gradual approach of the paraxial and marginal curves and the flattening of these curves as spherical and chromatic errors diminish. In general, within the range examined, as the focal length diminishes with increasing curvatures of the converging surfaces, there is less chromatic aberration and the marginal rays move from a state of overcorrection to one of undercorrection. An increase in curvature of the second (diverging) surface, has of course an exactly opposite result in all three respects. In order to make a comparison between the effects produced at each surface, the rate of change of curvature must be taken into account and this depends upon the part of the tool list from which the tool numbers are taken. Accordingly, the percentage increase of curvature of each surface from one lens to the next in the set and the percentage change of focal length produced thereby were calculated, together with the accompanying changes of chromatic and spherical aberration expressed as a percentage of the mean focal length. For spherical aberration, values for *D* rays were used, and as a measure of chromatic aberration, the separation along the axis between the extreme colours, which in most cases were *D* and *G'*, but in a few, *F* and *G'*. From these data, the corresponding changes for an alteration in curvature of 1 per cent., can be deduced. The results obtained are given below in tabular form.

Average changes for 1 per cent. increase in curvature

		Focal length per cent.	Spherical aberration percentage of mean focal length	Chromatic aberration percentage of mean focal length
Set I	1st surface	−1·20	−·025	−·032
	2nd ,,	+ ·24	+·027	+·028
Set II	1st ,,	− ·83	−·021	−·029
	2nd ,,	+ ·40	+·047	+·047
	3rd ,,	− ·45	−·015	−·011
Set III	1st ,,	−1·00	−·013	−·027
	2nd ,,	+ ·11	+·015	+·029

Another interesting and important observation to be made from the curves is the shifting of the point of intersection of paraxial and marginal curves from red to blue. This movement is roughly uniform, taking the direction from red to blue as the converging surfaces increase in curvature and from blue to red with increase in curvature of the diverging surfaces. Should any particular type of colour correction be desired, it would be quite useful to employ such sets of curves to determine the combination most suitable for the purpose.

The shape of the curves themselves and their inclinations to the vertical is of interest. The curvature of the marginal curve is always greater than that of the paraxial curve, approximately 20 per cent. greater. With the employment of different tools, the curvatures of both paraxial and marginal curves change. It seems, however, that they change at very nearly the same rate, their relative change of curvature being small compared with the mean change of curvature of either. The curves become flatter either with an increase in curvature of the first surface or with a decrease in curvature of the second surface. In other words, they become flatter as the colour for which spherical aberration is corrected moves from red to blue. In the same way, although the inclinations of both the paraxial and marginal curves change, yet examination of the results show that the angle between each pair is practically constant, that is, the variation in their relative inclination is very small. These results are of importance for they lead to the conclusion that when the paraxial and marginal curves are known for one type of colour correction, they can be deduced for any other type from a knowledge of the paraxial curve only.

The systems for which correction for spherical aberration has been obtained for *D*, as shown by the curves, agree in a very satisfactory way with the calculated form obtained by algebraical methods. In some cases there is more than one set of curves intersecting at *D*. The corresponding doublets differ very slightly from the calculated lens, generally by a difference of only 1 in the tool numbers. The results therefore justify the use of algebraical calculation without reckoning thicknesses and the selection of the nearest tool from the list.

Below are given the calculated curvatures for the three surfaces for each type with the nearest tool number which would naturally be used in their construction and, for comparison, the tool numbers and curvatures of those combinations corrected for *D*, as shown by the curves.

		Curvatures			Nearest tool numbers		
In Set I:	Calculated	9·520	9·318	·051	139	137	1*
	From curves	9·496	9·404	0	139	138	0
		9·588	9·496	0	140	139	0
		9·404	9·221	0	138	137	0
In Set II:	Calculated	5·918	15·289	3·555	97	195	64
	From curves	5·951	15·148	3·590	97	194	64
In Set III:	Calculated	6·888	12·126	·710	109	166	15*
	From curves	6·739	11·914	·777	107	164	16
		6·819	11·914	·777	108	164	16
		6·900	12·016	·777	109	165	16

* Before the trigonometrical calculations were made, rough values for the curvatures were obtained from the N.P.L. Tables already mentioned, without applying certain corrections. This led to the use of Tool 16 instead of 15 which subsequent calculation showed is given by algebraical method. The change however alters the focal length only, its influence on the spherical aberration being quite inappreciable. The same remark applies to the use of a flat surface instead of tool 1 in Set I.

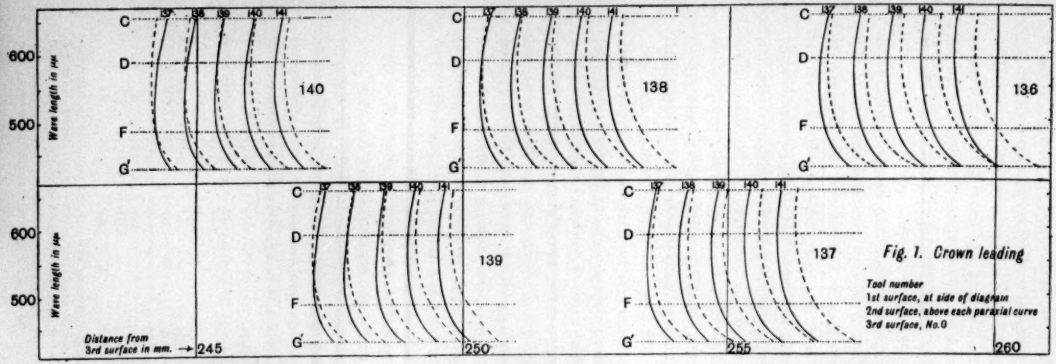


Fig. 1. Crown leading

Tool number
1st surface, at side of diagram
2nd surface, above each paraxial curve
3rd surface, No. 0

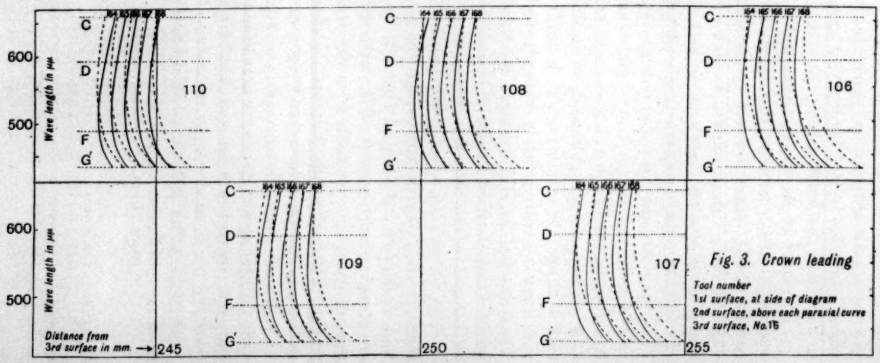
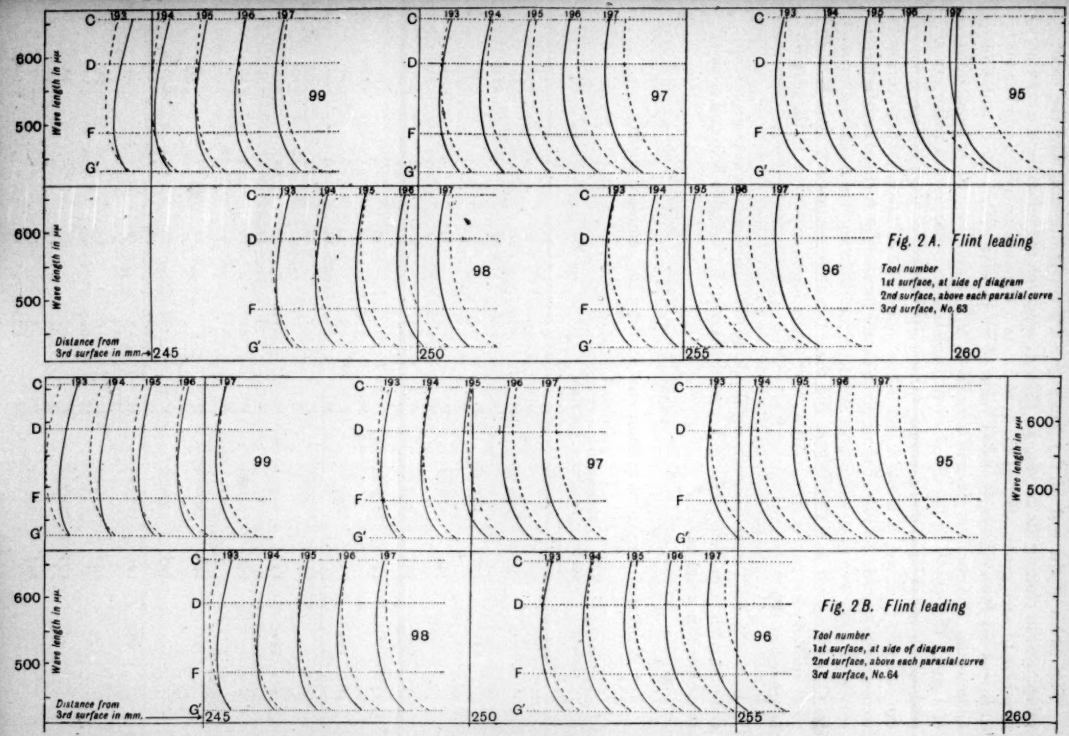


Fig. 3. Crown leading

Tool number
1st surface, at side of diagram
2nd surface, above each paraxial curve
3rd surface, No. 15

Figs. 1 and 3.



Figs. 2 A and 2 B.

The tables at the end of the paper give information concerning the exact values used in plotting the curves. In Table II are given the values for the extreme members of each group, correct to ·01 mm. As a standard reference number for each lens has been chosen the mean value of the paraxial results for C and F in millimetres. The numbers given in columns C, D, F, and G' express, in hundredths of a mm., the length to be added to the standard in order to obtain the exact readings on the paraxial and marginal curves. Intermediate curves in each group can easily be obtained, as it was found in the course of calculation that the curves were equally spaced. In Table III are given details for some of the most satisfactory curves. In only two cases given does the departure from the mean paraxial value for C and F exceed half a millimetre and these two show particularly close agreement and very flat curves from C to F, only separating to any extent in the neighbourhood of G'.

In conclusion, I wish to express my thanks to Mr T. Smith, at whose suggestion and under whose guidance this work was undertaken, for valuable help and advice throughout its progress.

Table II

Numerical results for extreme curvatures of the cemented surface

1st surface	2nd surface	Reference number	Paraxial or marginal	C	D	F	G'
Set I							
136	137	256·80	P	+1	−11	−1	+44
			M	+9	+2	+22	+91
	141	259·33	P	−10	−16	+10	+70
			M	+28	+25	+67	+146
137	137	253·58	P	+5	−8	−4	+39
			M	+8	−1	+13	+77
	141	256·05	P	−7	−14	+7	+63
			M	+22	+21	+59	+132
138	137	250·42	P	+6	−8	−6	+31
			M	+4	−7	+6	+62
	141	252·82	P	−4	−13	+4	+55
			M	+17	+14	+50	+116
139	137	247·32	P	+9	−5	−8	+26
			M	+2	−13	+0	+50
	141	249·66	P	−1	−11	+1	+49
			M	+16	+9	+44	+101
140	137	244·31	P	+11	−6	−11	+19
			M	−8	−17	−11	+33
	141	246·58	P	+2	−10	−1	+42
			M	+15	+4	+35	+82
Set II A							
95	193	256·79	P	+0	−11	+1	+48
			M	+5	−2	+28	+89
	197	260·06	P	−14	−18	+14	+80
			M	+27	+26	+80	+164
96	193	253·65	P	+3	−10	−4	+36
			M	+4	−7	+19	+72
	197	256·82	P	−9	−15	+10	+69
			M	+29	+26	+70	+147
97	193	250·53	P	+7	−8	−8	+26
			M	+0	−11	+3	+48
	197	253·63	P	−5	−14	+5	+57
			M	+23	+19	+53	+120

Table II (*continued*)

Numerical results for extreme curvatures of the cemented surface

1st surface	2nd surface	Reference number	Paraxial or marginal	*C*	*D*	*F*	*G'*
98	193	247·48	P	+12	−6	−12	+17
			M	−5	−20	−9	+29
	197	250·51	P	−1	−11	+1	+46
			M	+15	+10	+38	+99
99	193	244·48	P	+15	−5	−16	+6
			M	−13	−27	−30	+11
	197	247·43	P	+3	−10	−4	+36
			M	+8	+2	+20	+77
Set II B							
95	193	254·55	P	+1	−11	−1	+43
			M	+0	−9	+19	+76
	197	257·75	P	−12	−17	+13	+75
			M	+23	+26	+70	+151
96	193	251·45	P	+5	−9	−5	+32
			M	−1	−12	+10	+60
	197	254·57	P	−7	−14	+8	+64
			M	+22	+20	+60	+133
97	193	248·39	P	+9	−7	−10	+22
			M	−6	−16	−6	+35
	197	251·43	P	−3	−12	+4	+53
			M	+16	+14	+44	+104
98	193	245·39	P	+13	−5	−13	+13
			M	−11	−25	−19	+16
	197	248·37	P	+1	−10	−1	+42
			M	+9	+4	+28	+87
99	193	242·43	P	+17	−3	−17	+4
			M	−16	−36	−37	+0
	197	245·34	P	+5	−9	−5	+31
			M	+2	−5	+11	+65
Set III							
106	164	256·12	P	+4	−9	−5	+34
			M	+1	−4	+19	+72
	168	257·12	P	−4	−13	+5	+55
			M	+6	+4	+44	+113
107	164	253·00	P	+7	−8	−8	+26
			M	+0	−8	+13	+64
	168	253·98	P	−2	−12	+2	+47
			M	+5	+2	+38	+100
108	164	250·00	P	+9	−7	−10	+20
			M	+0	−9	+7	+50
	168	250·96	P	+2	−11	−2	+40
			M	+3	+1	+30	+88
109	164	247·02	P	+12	−5	−12	+14
			M	−3	−11	+0	+44
	168	247·95	P	+3	−9	−4	+34
			M	+4	−1	+23	+82
110	164	247·07	P	+14	−4	−14	+9
			M	−4	−16	−9	+31
	168	244·98	P	+6	−8	−6	+28
			M	+1	−7	+14	+67

Table III

Numerical results for selected objectives

	1st surface	2nd surface	Reference number	Paraxial or marginal	*C*	*D*	*F*	*G′*	Colour corrected wave-length
I	139	138	247·91	P	+6	−7	−7	+31	589
				M	+5	−8	+10	+63	
		137	247·33	P	+8	−6	−9	+25	530
				M	+1	−14	−1	+51	
	140	139	245·44	P	+6	−8	−6	+30	589
				M	+4	−6	+7	+56	
		138	244·88	P	+8	−8	−9	+24	550
				M	−3	−12	−3	+45	
II A	98	194	248·22	P	+9	−7	−9	+24	560
				M	+1	−11	+5	+48	
		193	247·48	P	+12	−6	−12	+17	500
				M	−5	−20	−9	+29	
	99	195	245·94	P	+9	−7	−10	+21	540
				M	−1	−11	−3	+45	
		194	245·20	P	+12	−6	−13	+14	480
				M	−6	−18	−16	+28	
		193	244·48	P	+15	−5	−16	+6	430
				M	−13	−27	−30	+10	
II B	98	194	246·14	P	+10	−7	−11	+20	500
				M	−6	−19	−8	+33	
		193	245·39	P	+13	−5	−13	+13	450
				M	−11	−25	−19	+16	
	99	195	243·88	P	+11	−5	−11	+18	470
				M	−6	−20	−13	+33	
		194	243·16	P	+14	−5	−12	+10	450
				M	−11	−28	−25	+16	
III	110	166	245·53	P	+10	−6	−11	+18	550
				M	−3	−12	+2	+48	
		165	244·30	P	+12	−5	−12	+13	530
				M	−4	−14	−4	+39	

APPENDIX

It has been suggested that empirical formulae fitting any of the curves in the series investigated might prove useful and further examination of the curves leads to the following results.

The equation of any curve of the series may be expressed in the form

$$y = a + bx + cx^2 \mp d,$$

where y represents the difference between the intersection distance for the particular ray and the reference number given in the table of numerical results (Table II), and x represents the coordinate for the wave-length in question. The values of x chosen for C, D, F and G' are -1, $-\frac{1}{2}$, $+\frac{1}{2}$, $+1$, which correspond to an origin midway between D and F, the four points being then, accurately enough, spaced equally on either side of this origin. The last term of the equation, $\mp d$, gives the departure from true parabolic form and throughout the series is to be taken equal to ∓ 4, being alternately $-$ and $+$ in the order C, D, F and G'. The

coefficients a, b, and c are measures respectively of the displacement from the reference point, the mean inclination and the curvatures of the curves. Corresponding to the constant difference between the inclinations of the paraxial and marginal curves and to the constant ratio of their curvatures, we have the values b' and c' for the marginal curves determined from the b and c of the paraxial curves by the relations $b' = b + \text{constant}$, $c' = \text{constant} \times c$.

Knowing the values of a, b, c and d for any paraxial curve and the corresponding values a', b', c' and d' for the marginal curve, we are asked to determine the equations which will give the shape of the curves if a departure of t dioptres is made from the power of any surface of the lens for which the curves are fully known.

The best equations of the different sets are as follows:

Variation of power at First Surface

I.	P.	$(a - 4t) + (b - 80t)\,x + (c - 30t)\,x^2 \mp d = y,$
	M.	$(a' - 110t) + (b - 80t + 15)\,x + 1{\cdot}3\,(c - 30t)\,x^2 \mp d = y.$
II A.	P.	$(a - 15t) + (b - 130t)\,x + (c - 50t)\,x^2 \mp d = y,$
	M.	$(a' - 150t) + (b - 130t + 15)\,x + 1{\cdot}1\,(c - 50t)\,x^2 \mp d = y.$
II B.	P.	$(a - 7t) + (b - 140t)\,x + (c - 50t)\,x^2 \mp d = y,$
	M.	$(a' - 180t) + (b - 140t + 15)\,x + 1{\cdot}15\,(c - 50t)\,x^2 \mp d = y.$
III.	P.	$a + (b - 80t)\,x + (c - 40t)\,x^2 \mp d = y,$
	M.	$(a' - 50t) + (b - 80t + 20)\,x + 1{\cdot}2\,(c - 40t)\,x^2 \mp d = y.$

Variation of power of Second Surface

I.	P.	$(a - 40t) + (b - 420t)\,x + (c - 160t)\,x^2 \mp d = y,$
	M.	$(a' - 860t) + (b - 420t + 15)\,x + 1{\cdot}15\,(c - 160t)\,x^2 \mp d = y.$
II A.	P.	$(a - 14t) + (b - 400t)\,x + (c - 160t)\,x^2 \mp d = y,$
	M.	$(a' - 800t) + (b - 400t + 20)\,x + 1{\cdot}2\,(c - 160t)\,x^2 \mp d = y.$
II B.	P.	$(a - 40t) + (b - 400t)\,x + (c - 140t)\,x^2 \mp d = y,$
	M.	$(a' - 780t) + (b - 400t + 20)\,x + 1{\cdot}15\,(c - 140t)\,x^2 \mp d = y.$
III.	P.	$(a - 70t) + (b - 920t)\,x + (c - 330t)\,x^2 \mp d = y,$
	M.	$(a' - 920t) + (b - 920t + 22)\,x + 1{\cdot}2\,(c - 330t)\,x^2 \mp d = y.$

From an examination of these results it seems that a single paraxial equation may be made to fit all the sets. For marginal results the correspondence is not so close but the same equation might be used for Sets I, II A and B, without any very great increase of inaccuracy. The equations for the two surfaces are too widely different for a compromise to be possible.

Paraxial Equations for all Sets:

1st Surface varied $(a - 10t) + (b - 120t)\,x + (c - 40t)\,x^2 \mp d = y,$

2nd „ „ $(a - 30t) + (b - 400t)\,x + (c - 160t)\,x^2 \mp d = y.$

Marginal Equations for Sets I, II A and B:

1st Surface varied $(a' - 150t) + (b - 120t + 15)\,x + 1{\cdot}2\,(c - 40t)\,x^2 \mp d = y,$

2nd „ „ $(a' - 820t) + (b - 400t + 18)\,x + 1{\cdot}2\,(c - 160t)\,x^2 \mp d = y.$

The equations of paraxial curves of the calculated lenses in each set and the value of a' for the marginal curves are as follows:

	Paraxial curve	Value of a'
I.	$-14{\cdot}5 + 4{\cdot}7x + 32x^2 \mp 4 = y$	$-17{\cdot}3$
II A.	$-15{\cdot}1 + 16{\cdot}6x + 36{\cdot}7x^2 \mp 4 = y$	$+5{\cdot}5$
II B.	$-15{\cdot}7 + 8{\cdot}3x + 33{\cdot}5x^2 \mp 4 = y$	$-10{\cdot}0$
III.	$-15{\cdot}3 - 5{\cdot}2x + 31x^2 \mp 4 = y$	$-7{\cdot}2$

It is important to know what errors in the determination of y the use of these equations is likely to cause. Within the range of variations in power investigated, that is, about ·2 dioptres for the first surface and ·03 dioptres for the second surface, when the best equation is used for each set, it was found that the error might amount to ± 10 units, while for the more general equations, the error was not much greater, being always less than ± 15 units. These are extreme cases and as a general rule the error may be expected to be much less, by far the greater number of results being within ± 5 units of the correct result. When it is remembered that the unit is ·01 mm. it will be seen that this degree of accuracy is probably good enough for many purposes.

Summary. Investigations on series of cemented telescope objectives having surfaces of slightly varying curvatures have shown that the results for the intersection lengths of paraxial and marginal rays may be plotted as curves which show striking uniformity throughout the series. The shape of the curves is very nearly parabolic and examination of their varying inclinations and curvatures shows that the angle between the paraxial and marginal curves is practically constant, while the curvature of the marginal curve is greater than that of the paraxial curve in a constant ratio. These results lead to the conclusion that the curves for any lens in the series can be predicted from a knowledge of both paraxial and marginal curves of one other form. The best results correspond closely with the objective determined by algebraic calculations in which thickness is neglected.

DISCUSSION

Instructor-Commander T. Y. Baker congratulated Miss Gillman on being the first lady to read a paper before the Society. The paper was likely to be very useful, but an examination of the curves suggested that simple relations should exist between the curves and the changes in the shapes of the lenses. Such a form, if possible, would be the best one in which the results obtained could be expressed.

Mr T. Smith suggested that as Miss Gillman had obtained results such as Commander Baker mentioned, they should be appended to the paper.

ON THE CORRECTION OF FIRST ORDER ASTIGMATISM OF A SINGLE LENS USED WITH A STOP

By Instructor-Commander T. Y. BAKER, R.N., B.A.

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In a paper read at the Optical Society on May 9th, 1918, by Major L. N. G. Filon and the present author, a value was given of the parameter of the first order caustic when the image point was on the axis of the lens. The range of image positions, for which the correction of first order spherical aberration is theoretically possible, was obtained and shown to be approximately coincident with the range of positions for which the lens would need to be of very steep meniscus form. It appeared consequently that over-correction of spherical aberration with a single lens was only possible for an extremely limited range of image positions and that the amount of over-correction could only be minute.

In this present paper a somewhat similar investigation is made of the positions for which correction of first order astigmatism is possible when there is choice of the shape of the lens and of the position of the stop. A short investigation is also made of the necessary extension to a system of lenses.

Preliminary Formulae

A thin lens of refractive index n and focal length f with air on both sides, has its back principal plane at AP (Fig. 1) and is used so as to produce an image under the Gaussian notation at IQ. Let WP, one of the rays proceeding from a certain point in the object plane, be a ray which would, if unaberrated, proceed to the

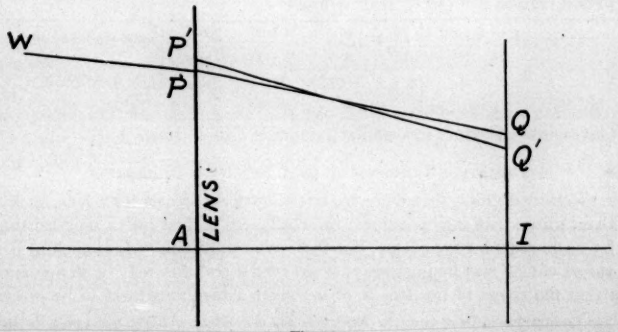

Fig. 1.

point Q in the image plane so that the Gaussian ray path is WPQ. Actually the refraction of the ray at the two surfaces of the lens alters the direction and the position of the emergent ray so that it reaches the image along the line $P'Q'$ and it is assumed that the aberration is sufficiently closely expressed by retaining first order terms only.

Let l_1 and l_2 be the intercepts of the Gaussian ray path upon planes AP and IQ. Then it can be shown that

$$QQ' = \frac{l_1^3 F(1-M)^2}{2v}\left[\frac{n+2}{n}\left\{\frac{C}{F} - \frac{n+1}{n+2}\cdot\frac{1+M}{1-M}\right\}^2 + \frac{n^2}{4(n-1)^2}\left\{1 - \frac{(n-1)^2}{n(n+2)}\left(\frac{1+M}{1-M}\right)^2\right\}\right] + \frac{3l_1^2 l_2 F(1-M)}{2v}\left[\frac{n+1}{n}\left\{\frac{C}{F} - \frac{2n+1}{2n+2}\cdot\frac{1+M}{1-M}\right\}\right] + \frac{l_1 l_2^2 F}{2v}\left[3 + \frac{1}{n}\right] \quad ..(1),$$

where F is the power of the lens, M the linear magnification of the image, v the distance of the image plane from the lens and C/F denotes the shape of the lens, that is, the ratio of the mean curvature of the surfaces to the power.

When the lens is used with a stop the consideration of the curvature of field, etc., demands that the ray shall be specified in terms of the intercept on the stop rather than on the lens. The necessary relationship between l_1 and λ the intercept on the stop is

$$l_1 = (v\lambda - v'l_2)/(v - v'),$$

where v' is the distance of the stop from the lens. Substituting this value, we obtain for the lateral aberration on the image plane

$$QQ' = \lambda^3 A + \lambda^2 l_2 B + \lambda l_2^2 C + l_2^3 D,$$

where A, B, C, D are all functions of v, v' and C/F the lens shape, each of the coefficients being a quadratic function of C/F.

The astigmatism term $\lambda l_2^2 C$ leads in the usual way to a definite curvature for the locus of primary foci for a given zone of the lens.

This curvature is $F(3k' + 1/n)$* where

$$k' = \frac{n+2}{n}\left\{\frac{vv'F}{v-v'}\left(\frac{C}{F}\right) - \frac{n+1}{n+2}\left(\frac{v+v'}{v-v'} - \frac{vv'F}{v-v'}\right)\right\}^2 - \frac{n}{4(n+2)}\left\{\frac{v+v'-vv'F}{v-v'} - \frac{n+2}{n}\right\}^2 + \frac{n^2}{4(n-1)^2}\left\{\frac{vv'F}{v-v'}\right\}^2 \quad ..(2).$$

The sagittal curvature in a similar manner is $F(k' + 1/n)$.

Astigmatism Coefficient of a Specified Magnitude

It therefore appears that there are two theoretical shapes for a lens, such that for given image and stop positions, the coefficient of first order astigmatism, k', can be made anything we please, but there are complications inasmuch as (i) the two values of C/F may be imaginary, or (ii) even if real they may be so numerically large that the shape of the lens is of too steep a meniscus form to be practical.

* See Coddington's *Treatise on the Reflection and Refraction of Light*, pp. 136–7, or Dennis Taylor, *System of Applied Optics*, pp. 150–1.

Looked at from another point of view the equation (2) indicates, for given image and stop positions and given shape of lens, the curvatures of the curves of meridional and sagittal curvature, the values of which are $F(3k'+1/n)$ and $F(k'+1/n)$. When $k'=0$ both these curves coincide with the Petzval curve; when k' is negative they lie to the right of it. Hence for flattening the field or for making it curve in the opposite sense, we must look for the smallest positive or largest negative value of k'.

The equation can be expressed

$$k' = \frac{n+2}{n}\cdot(\text{positive quantity}) - \frac{n}{4(n+2)}\cdot(\text{positive quantity}) + \frac{n^2}{4(n-1)^2}(\text{positive quantity}).$$

The last two of these three positive quantities depend on the image and stop positions only; the first one depends in addition on the shape of the lens. With the lens shape at our disposal the first one can be made zero provided the shape of the lens is not thereby rendered too steep a meniscus, which means practically that C/F must lie between $+4$ and -4. The other two terms then give the limit to which flattening of the field can be obtained.

Graphical Representation

In order to simplify the relationships expressed by (2) and show them in a graphical form a series of curves of constant k' has been constructed in the following manner (see Fig. 2).

Write

$$\frac{1}{2}\sqrt{\frac{n}{n+2}}\left\{\frac{v+v'-vv'F}{v-v'}-\frac{n+2}{n}\right\} \text{ as } x, \text{ and } \frac{1}{2}\frac{n}{n-1}\left\{\frac{vv'F}{(v-v')}\right\} \text{ as } y.$$

From these we obtain by reduction

$$\frac{1}{vF}=\frac{x\sqrt{n(n+2)}+y(n-1)+1}{2(n-1)y} \qquad \ldots.(3)$$

and

$$\frac{1}{v'F}=\frac{x\sqrt{n(n+2)}+y(n-1)+n+1}{2(n-1)y} \qquad \ldots.(4).$$

Equation (2) now becomes

$$\frac{4(n+1)^2}{n^3(n+2)}\left\{x\sqrt{n(n+2)}+\tfrac{1}{2}(n+2)-\frac{(n+2)(n-1)}{n+1}\frac{C}{F}y\right\}^2-x^2+y^2=k' \quad ..(5).$$

Now equations (3) and (4) indicate the relationships between x and y of the diagram and v and v' which are the image and stop distances from the lens. C/F being still at our choice it is selected so as to make

$$x\sqrt{n(n+2)}+\tfrac{1}{2}(n+2)-\frac{(n+2)(n-1)}{(n+1)}\frac{C}{F}y=0,$$

providing this can be done without exceeding the numerical limits for C/F. For a certain part of the diagram consequently (and the limits to this part must be investigated later), the constant astigmatism curves reduce to

$$-x^2+y^2=k'$$

a series of rectangular hyperbolas with $x=\pm y$ as asymptotes.

SCALE OF LENS SHAPES
SCALE OF STOP POSITIONS
SCALE OF IMAGE POSITIONS
SCALE OF LENS SHAPES
Scale of additional Astigmatism Coeffts. when Petzval Curve is not flat
A
B
C
O
Z
c/r = -4.0
c/r = -2.0
c/r = -1.0
c/r = 0.0
c/r = +1.0
c/r = +2.0
c/r = +4.0

Fig. 2.

Image and Stop Positions in Diagram

Values of v and v' can be shown in the diagram by means of equations (3) and (4) for we shall have from (3)

$$x\sqrt{n(n+2)} + y(n-1)(1-2/vF) + 1 = 0 \qquad \ldots.(6),$$

which is a straight line as long as v remains constant and becomes a pencil of lines through $-1/\sqrt{n(n+2)}$, 0 when v is treated as a variable parameter.

Similarly we obtain from (4)

$$x\sqrt{n(n+2)} + y(n-1)(1-2/v'F) + n + 1 = 0 \qquad \ldots.(7),$$

a pencil of lines through $-(n+1)/\sqrt{n(n+2)}$, 0 every one of which corresponds to a definite value of v'.

When $v' = \infty$ equation (7) becomes

$$x\sqrt{n(n+2)} + y(n-1) + n + 1 = 0 \qquad \ldots.(8),$$

and this line cuts (6) where $y = -nvF/2(n-1)$. In other words the pencil of lines (6) cuts the line (8) in a constant scale of v, and in a precisely similar manner it may be shown that the pencil of lines (7) cuts the line $v = \infty$ in a constant scale of v', and that the scale values of v and v' are equal.

Lens Shapes in Diagram

The values of C/F which give the greatest possible flattening of the field can also be represented by a pencil of lines, for we have, in such case,

$$x\sqrt{n(n+2)} - \frac{(n+2)(n-1)}{n+1}\frac{C}{F}y + \tfrac{1}{2}(n+2) = 0 \qquad \ldots.(9),$$

a pencil of lines through $-\frac{1}{2}(n+2)/\sqrt{n(n+2)}$, 0. This point is mid-way between the centres of the v and v' pencils. C/F can therefore be shown as a constant scale along any line parallel to Ox. This scale for C/F has been drawn close to the top and bottom of the diagram so as to make it as open as possible.

The lines $C/F = \pm 4$, which have been arbitrarily fixed as the limits of form beyond which a lens cannot conveniently be made, lie in directions approximately N.E. and N.W. and it is now clear that the area of the figure for which $-x^2 + y^2 = k'$ can be taken as the constant astigmatism curves is limited to the sectors N.W. to N.E. and S.E. to S.W. For the rest of the figure we can only allow the limiting value of C/F, viz. $C/F = 4$ from N.E. to E., $C/F = -4$ from E. to S.E., $C/F = 4$ from S.W. to W. and $C/F = -4$ from W. to N.W.

Modified Astigmatism Curves

With these modifications the constant astigmatism curves can still be drawn but will no longer be rectangular hyperbolas. For example, in the N.E. to E. sector they will be the conics

$$\frac{4(n+1)^2}{n^3(n+2)}\left\{x\sqrt{n(n+2)} + \tfrac{1}{2}(n+2) - \frac{4(n+2)(n-1)}{n+1}y\right\}^2 - x^2 + y^2 = k',$$

a series of hyperbolas which touch the rectangular hyperbolas at the points where they cross the line $C/F = 4$. The three other sectors can be treated in a similar way.

It is clear from what has been said above that the curves of constant k' have no break in them (in technical language there is no "point d'arrêt"), but have discontinuities of direction where they cross $y = 0$ and discontinuities of curvature, but not of direction, where they cross $C/F = \pm 4$.

It is thought that this set of curves, which has been plotted for the value $n = 1{\cdot}518$, will be of assistance to computing opticians in determining approximately the greatest flattening of field that is possible for given image and stop positions and the shape of the lens that will do it.

Method of using Astigmatism Diagram

The position on the diagram corresponding to the image and stop positions has to be first determined. The image position corresponds to a line through the centre A of the stop position scale and the particular value of vF corresponding to the image position. For example, if the image is formed at half the focal distance beyond the lens, the image position line is the pecked line AZ.

The stop position line is drawn in a similar way from the centre B of the image position scale and the point on the stop position scale corresponding to the value of $v'F$. In the example the stop is taken to be a virtual one $-3f$ beyond the lens and the stop position line is BZ intersecting the image position line in Z.

Note.—For both image and stop, the values of vF and $v'F$ are those corresponding to the positions after refraction at the lens, not before. For the stop position above, the image is virtual and $v'F$ being -3 puts this virtual image to the left of the lens. It is this value which has to be used, not the value $-\frac{3}{4}$ which is the distance of the real stop from the lens before refraction.

The value of k' can then be taken from the diagram, viz. $-0{\cdot}3$.

The best shape of the lens is found by running a line CZ to the lens shape scale. It gives for this example, $C/F = 0{\cdot}50$ as the best lens shape.

Any other lens shape will add a positive quantity to k'. Its value is

$$4\,(1 + 1/n)^2 a^2,$$

where a is the intercept between the point corresponding to the image and stop positions and the C/F line which corresponds to the lens shape that is used, such intercept being measured parallel to the line Ox. The additional astigmatism term can therefore be obtained immediately by measuring this intercept against a scale so constructed that it will square it and multiply it by the requisite factor $4\,(1 + 1/n)^2$. Such a scale is drawn in Fig. 2 on the Ox line.

As an illustration let the lens considered above, for which the best shape is $0{\cdot}50$, be replaced by one in which the shape is $-1{\cdot}50$. By measuring the intercept ZM against the special scale drawn along Ox we find the extra astigmatism coefficient to be $1{\cdot}7$ so that this change in lens shape has altered the coefficient from $-0{\cdot}3$ to $+1{\cdot}4$.

Deductions from Astigmatism Diagrams and Formulae

Certain deductions can be made from the value of k' as given in equation (2) and as plotted.

The form of the lens being chosen so as to make the first term zero involves that

$$\frac{C}{F}=\frac{n+1}{n+2}\left(\frac{1}{vF}+\frac{1}{v'F}-1\right).$$

From this we see that the lens will still give flattest field if image and stop positions be interchanged. The lens shape is equi-convex (or equi-concave) if

$$1/v+1/v'=F.$$

When the best form has been chosen (C/F being between ± 4) the value of k' is given by

$$k'=\frac{-n}{4(n+2)}\left\{(1+M)\frac{v'}{v-v'}-\frac{2}{n}\right\}^2+\frac{n^2}{4(n-1)^2}\left\{\frac{(1-M)v'}{v-v'}\right\}^2$$

which reduces to

$$k'=\frac{-n}{4(n+2)}\left\{\frac{v+v'}{v-v'}-\frac{vv'F}{v-v'}-\frac{n+2}{n}\right\}^2+\frac{n^2}{4(n-1)^2}\left\{\frac{vv'F}{v-v'}\right\}^2,$$

whence it follows that an interchange of image and stop positions only preserves the same value of k' provided

$$v+v'=vv'F$$

or

$$1/v+1/v'=F.$$

Such interchange of image and stop positions consequently involves also that the lens should be of "Equi" form.

These are the same results, obtained in a different manner, as those presented by Mr Chaundy in a paper recently read before the Optical Society.

If the stop position is at the lens, then for all image positions $k'=1$.

If the image position is at the lens then for all stop positions $k'=0$.

The curve of positions for which $k'=0$ is seen in the figure. In the S.W. to W. sector, this curve is a portion of a hyperbola lying very close to its asymptote so that it is almost a straight line. It will be seen that this approximate straight line corresponds very nearly to $v'=0{\cdot}23$ and that if the image is formed anywhere between the stop position and the lens, the field will coincide with the Petzval curve provided the lens have the steep meniscus form $C/F=4$. A similar statement can be made respecting a virtual stop position at $v'=-0{\cdot}30$. In this case the image is also virtual and the lens is reversed. In fact the whole system is the same as before with the *light* reversed.

To obtain *large* negative values of k' there is only a very small choice as to the image and stop positions.

The value of v must be about $0{\cdot}35$ and v' about $0{\cdot}45$, k' becoming numerically greater as image and stop move towards one another. The change in k' takes place very rapidly for quite small changes in v or v'. The suitable form is the meniscus $C/F=4$.

On the other hand when $v = 0{\cdot}15$ and $v' = 0{\cdot}10$ very big positive values of k are inevitable. These again are least with the extreme meniscus form.

If the image position is about 0·53, the stop position can be varied very considerably without affecting the value of k', which will remain about $-0{\cdot}5$. The lens shape however must be changed at the same time.

Extension to Lens Systems

So far, the astigmatism of a single lens only has been considered, but the results are easily extended to a series of lenses along an axis.

In Fig. 3 an image is formed on the curved surface I_0T_0 whose curvature is B_0. The Gaussian image of T_0 in the next lens is T_2. Using the ordinary notation we have

$$1/v_0 = 1/v_2 + F_1.$$

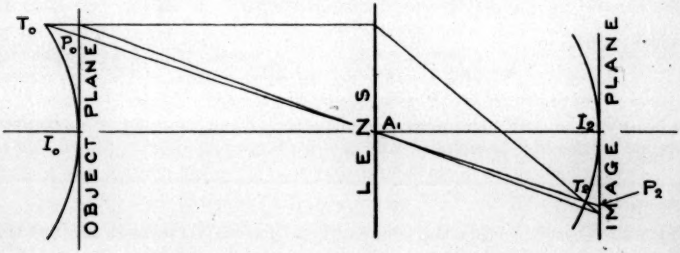

Fig. 3.

Now P_0, being a point at small distance from T_0, will have an image P_2 at small distance from T_2, $T_0P_0A_1T_2P_2$ being a straight line.

Within the order of accuracy with which we are dealing we can write

$$P_0T_0 = \delta v_0, \quad P_2T_2 = \delta v_2,$$

and clearly

$$\delta v_0/v_0^2 = \delta v_2/v_2^2.$$

Further $T_0I_0/v_0 = T_2I_2/v_2$, from which it follows that T_0I_0 and T_2I_2 have the same curvatures. But this result is based upon a *Gaussian* transference from one medium to the next. If we include also the first order aberrations there must be imposed in addition the curvature of field produced by the lens.

Starting with a flat field and dealing with the meridional rays the first lens produces a curvature

$$F_1(3k_1' + 1/n).$$

This is transferred through the next lens and increased by $F_3(3k_3' + 1/n_3)$ so that the curvature is now

$$F_1(3k_1' + 1/n_1) + F_3(3k_3' + 1/n_3).$$

Finally, the curvature of the final meridional surface is

$$3\Sigma F_1k_1' + \Sigma F_1/n_1.$$

The sagittal curvatures follow in the same way and we have for the final result

$$\Sigma F_1 k_1' + \Sigma F_1/n_1.$$

Hence in order to obtain a flat field and freedom from astigmatism we must have the two conditions satisfied,

$$\Sigma F_1 k_1' = 0,$$
$$\Sigma F_1/n_1 = 0,$$

which are, of course, merely two of the five von Seidel conditions.

Design of Separated Systems

In dealing with a separated system in which the main lines of the design have been carried out according to the Gaussian scheme, coupled perhaps with the conditions of achromatism, there remain to the computer only the shapes of the lenses that can be varied in order to eliminate aberrations. Now, of the five first order defects, two are under investigation here, viz. absence of astigmatism which involves $\Sigma F_1 k_1' = 0$ and flatness of field which involves $\Sigma F_1/n_1 = 0$. The latter condition obviously cannot be satisfied if positive lenses only are used, but the first one may be satisfied provided the system is so arranged as to the various stop and image positions that some of the lenses, at all events, have negative values of k'. Positive values of k' more commonly occur and the computer's object is generally to reduce the positive terms as low as possible and when negative terms do occur to make the most of them. In other words his investigation is almost always to find a lens which will satisfy the condition that

$$\frac{C}{F} = \frac{n+1}{n+2}\left(\frac{1}{vF} + \frac{1}{v'F} - 1\right).$$

If this condition is satisfied he obtains a *stationary* value for k' for that lens and consequently a small departure from this condition produces a much smaller variation in the corresponding k'.

The diagram will enable him to determine in which lenses, if any, a considerable variation from the best shape is allowable without producing a serious increase in the field curvature. It is evident that he can afford to allow the astigmatism coefficient to increase if he is dealing with a low power lens whereas with a high power one a departure from the best shape has a much more serious effect.

Two Thin Lenses in Contact

A special case occurs, and merits further investigation, when two thin lenses are in contact, and are made of the same glass.

Let their powers be F_1, F_3 and the image and stop positions v_1, v_3; v_1', v_3'.

Now we have

$$1/v_3 = 1/v_1 + F_3,$$
$$1/v_3' = 1/v_1' + F_3,$$

whence

$$\frac{v_1 v_1'}{v_1 - v_1'} = \frac{v_3 v_3'}{v_3 - v_3'}.$$

Also

$$F_1k_1' = \frac{-n}{4(n+2)}\left\{\frac{v_1+v_1'}{v_1-v_1'} - \frac{v_1v_1'F_1}{v_1-v_1'} - \frac{n+2}{n}\right\}^2 F_1 + \frac{n^2}{4(n-1)^2}\left(\frac{vv'}{v_1-v_1'}\right)^2 F_1^3$$

$$= \frac{-n}{4(n+2)}\left\{\frac{v_3+v_3'}{v_3-v_3'} - \frac{v_3v_3'(F_1+F_3)}{v_3-v_3'} - \frac{n+2}{n} - \frac{v_3v_3'F_3}{v_3-v_3'}\right\}^2 F_1 + \frac{n^2}{4(n-1)^2}\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 F_1^3.$$

Similarly

$$F_3k_3' = \frac{-n}{4(n+2)}\left\{\frac{v_3+v_3'}{v_3-v_3'} - \frac{v_3v_3'(F_1+F_3)}{v_3-v_3'} - \frac{n+2}{n} + \frac{v_3v_3'F_1}{v_3-v_3'}\right\}^2 F_3 + \frac{n^2}{4(n-1)^2}\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 F_3^2.$$

Adding the astigmatism curvatures we obtain after reduction

$$F_1k_1' + F_3k_3' = \frac{-n}{4(n+2)}A^2(F_1+F_3) + \frac{n^2}{4(n-1)^2}\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 (F_1+F_3)^3 - F_1F_3(F_1+F_3)\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 \frac{n(2n+1)^2}{4(n+2)(n-1)^2},$$

where A stands for $\dfrac{v_3+v_3'}{v_3-v_3'} - \dfrac{v_3v_3'(F_1+F_3)}{v_3-v_3'} - \dfrac{n+2}{n}$.

Now if, instead of two lenses of powers F_1 and F_3, placed in contact and each of such curvature as to produce flattest field, a single lens of power F_1+F_3 had been used, its shape also being determined for the conditions that the field should be flattest, the astigmatism curvature would have been

$$(F_1+F_3)k' = \frac{-n}{4(n+2)}A^2(F_1+F_3) + \frac{n^2}{4(n-1)^2}\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 (F_1+F_3)^3.$$

With the two lenses, each of "best form," there is therefore a reduction in the astigmatism curvature represented by

$$\frac{n(2n+1)^2}{4(n+2)(n-1)^2}\left(\frac{v_3v_3'}{v_3-v_3'}\right)^2 F_1F_3(F_1+F_3).$$

This reduction is clearly greatest when F_1 and F_3 are equal, so that the object that we are seeking to attain is always brought nearer by splitting up a lens into two (each of half power) with the "best form" used in each case.

The numerical value of this reduction is obviously dependent on the image and stop positions and it is not possible at the present stage to make a thorough investigation, but a numerical example will illustrate to what an extent this reduction can be made.

Take the case of the image being at the back focus of a lens of power F, the stop being at a distance $4f$ beyond the lens.

Using the diagram we obtain $Fk' = 3{\cdot}1F$ as the best that can be done for the single lens.

For the two lenses each of power $F/2$ we have for the total astigmatism curvature $0{\cdot}14F$ a reduction of nearly $3F$, which checks with the reduction formula above.

This procedure might be of great use to the computing designer for, if he finds that one lens is contributing seriously to the astigmatism defect, he may be able to reduce the contribution very much by the simple expedient of replacing it by

two half power lenses in contact, since this change alters nothing as regards the Gaussian and chromatic dispositions. It does, however, introduce two more glass air surfaces and where illumination of the field is of importance it may not be possible to make the division owing to loss of light.

Ramsden and Huyghens Eyepieces

We conclude the paper with the investigation into the astigmatism defects of the Ramsden and Huyghens eyepieces. It will be assumed that the eyepiece is being used with a very long focus objective and the light is assumed to travel from the eye end. We are then investigating the curvatures of field at the back focus of the system, the final stop position being considered to be infinity.

The lenses are taken to be each of power F and separation $\frac{2}{3}f$.

We have

$$v_3F = \tfrac{1}{4}, \qquad v_3'F = \infty,$$
$$v_1F = 1, \qquad v_1'F = -\tfrac{1}{3}.$$

Making each lens of best astigmatic form, we have

$$k_3'F = -0{\cdot}88F, \qquad k_1'F = -0{\cdot}32F.$$

The meridional and sagittal curvatures are therefore $-2{\cdot}94F$ and $-{\cdot}54F$.

In the case of the Ramsden eyepiece it therefore appears that there is no need to aim at the best astigmatic form for either lens and considerable divergence from this best form is necessary, otherwise the astigmatism will be over-corrected.

In the Huyghens eyepiece for which the field lens is of power F, the eye lens $3F$ and the separation $\frac{2}{3}f$ we have

$$v_3F = \tfrac{1}{2}, \qquad v_3'F = \infty, \qquad k_3' = -1{\cdot}01,$$
$$3v_1F = 1, \qquad 3v_1'F = -1, \qquad k_1' = 0{\cdot}17.$$

The astigmatism curvature is therefore $-0{\cdot}50F$. Here again it will be seen that taking the best form for each lens we have over-corrected astigmatism, and a departure from the best form must be made.

DISCUSSION

Professor L. N. G. Filon considered that the author's ingenious chart would prove very useful to designers. He had found graphical methods such as the one suggested susceptible of high accuracy—in favourable cases they can be read to one-thousandth of an inch. Instead of drawing lines in pencil in using the chart he proposed that thin straight lines in ink on narrow strips of tracing paper should be employed, arranged to pivot about the centres of projection. The accuracy in reading was if anything increased by this method, and the drawing was also left clean.

Mr T. Smith said that, instead of a diagram, the calculations required could be reduced to simple addition, in finding the best lens form and the constants of any thin lens however built up, by the use of a table giving X in terms of x where $X = \frac{1+x}{1-x}$. In dealing with astigmatism two such quantities had to be added, x in the one case corresponding to the magnification of the stop and in the other case to the magnification of the image.

ABSTRACT

"A New Box Sextant." By C. PULFRICH, *Z. f. Instrk.* 39 (1919), 201.

The instrument is designed for hand use where it is not convenient to employ a theodolite with stand. It is adapted for the measurement of angles on the sea, in the air, or on land, both by night and day.

The instrument consists essentially of two prisms P_1, P_2 (Figs. 1 and 2), placed one behind the other. Fig. 1 shows the arrangement of the prisms as seen from above, and Fig. 2 represents a side view. P_1 is a simple right-angle prism with the mirror surface S_1, while P_2 is a double prism built up of two right-angle prisms, the two surfaces in contact at S_2 being both silvered. The prism P_1 is fixed relative to the apparatus. The prism P_2, together with the divided circle T and the axis A, can be rotated by means of both rough and fine adjustments. A third prism P_3 is built up of two parts, the surface of separation S_3 being half-silvered. The object of this prism is to enable the observer to get a free view over the prisms P_1 and P_2 as well as a view through them.

The chief advantage of using a prism system instead of an ordinary plane mirror is that the effective aperture remains more uniform during a rotation, as can be seen from the following table:

Effective apertures at different angles

	Plane mirror	Single prism mirror	Double prism mirror
0°	1	·5	·5
90° and 270°	·7	·7	·7
180°	0	·25	·5

In this table 0° represents the direction in which the light is reflected backwards. The values of the effective apertures of the prism systems are calculated on the assumption that the angles next the reflecting surfaces are each 45° and the refractive index of the glass is 1·57.

A further advantage of using a double prism instead of a single one is that for the complete circumference only one-half revolution of the prism is required.

It is necessary that the reflecting surfaces of the double prism should fit exactly together and should be parallel to each other, and further the prisms should be free from pyramidal errors. The angles next the reflecting surfaces need not be 45°, but in each prism they must be equal.

In order to be able to make measurements of angular distances greater than 135°, a pentagonal prism can be inserted in the path of the ray which enters the top of the prism P_3, so that the free view is now to the side.

The divided circle is viewed by means of a small microscope. The reading is accurate to about 1 minute.

The instrument is provided with a handle which can be screwed into the bottom or side of the box, according as the angle measurements are required in a horizontal or a vertical plane.

For observations on the sun a neutral tinted glass is provided for insertion between the prisms P_1 and P_3.

The method of obtaining an artificial horizon is as follows. A collimator, attached to

a pendulum system, is placed in front of the upper portion of the prism P_3. It can be adjusted until its optical axis is horizontal when the pendulum is at rest. In the field of view of the collimator there is a bright line on a dark background. By rotating the double

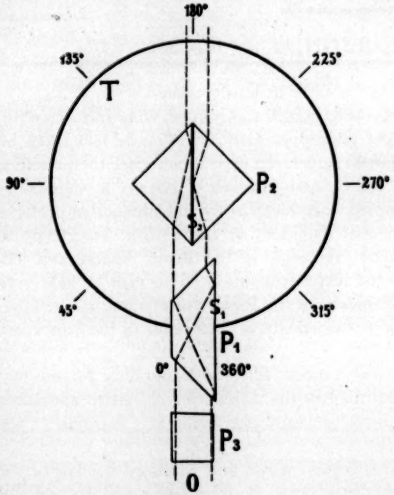

Fig. 1.

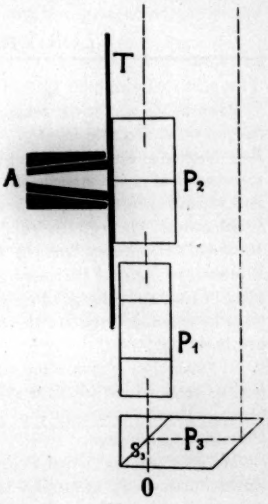

Fig. 2.

prism the image of a given object can be brought into the position where the displacements of the line on either side are equal, when the pendulum is swinging.

A small telescope (4 ×) may be used with the instrument.

The author describes how the apparatus is used in making a number of distinct types of measurements, and he suggests that it might be employed for determining the distances or sizes of distant objects.

J. S. A.

OBITUARY NOTICE

LORD RAYLEIGH, O.M., F.R.S.

THE principal events of LORD RAYLEIGH'S life are easily related.

He was the son of the second Baron Rayleigh, was born in 1842 and succeeded his father in the title in 1873. Eight years previously, in 1865, he had taken his B.A. Degree at Cambridge, as Senior Wrangler. In 1879 he accepted the invitation to become Maxwell's successor in the Cavendish Chair of Physics at Cambridge and, for five eventful years, was occupied with researches of fundamental importance to Electrical Theory. In 1885 he was President of the British Association at Montreal; two years later he became Secretary of the Royal Society and also Professor of Natural Philosophy in the Royal Institution. This post he held until 1905. From 1905 to 1908 he was President of the Royal Society and, in the latter year, he succeeded the fourth Duke of Devonshire as Chancellor of the University of Cambridge.

He was closely connected with the National Physical Laboratory for the first twenty years of its life, first as Chairman of the Treasury Committee appointed to consider the desirability of its establishment, afterwards as Chairman of the Executive Committee.

He received the Nobel Prize, the Copley, Royal and Rumford Medals of the Royal Society and numerous other distinctions, both at home and abroad. In 1905 he became a Privy Councillor and he was one of the first members of the Order of Merit.

His papers, some 400 in number, cover the whole range of Mathematical Physics. Many of them are fundamental in their character; there is not one which has not advanced knowledge in some degree. It is impossible, in the short space of this Notice, to attempt any full account of his work and it is with his Optical Papers that members of the Optical Society are chiefly concerned.

He was elected an Honorary Fellow of our Society in 1907. One of his last contributions to Science appears in the February number of our *Transactions*, "On the possible disturbance of a Range-Finder by Atmospheric Refraction due to the motion of the ship which carries it," and, until a brief period before his death, he was working at the colours of beetles' wings, a subject which had long interested him.

His optical work began quite early. Much of it was published in the *Philosophical Magazine* and the writer has a small octavo volume "From the Author," containing some twenty papers which appeared between 1871 and 1887 and form a store of most valuable information. The earlier papers deal with the scattering of light from small particles and lead to the now well-known explanation of the colour and polarisation of the light from the sky. The light is scattered by the

small particles in the atmosphere; this scattered light is partially polarised, while its intensity varies inversely as the fourth power of the wave-length. Hence the blue rays of sunlight are scattered much more copiously than the red; the clear sky is, therefore, blue in colour. These theoretical conclusions have recently been illustrated in a beautiful way by Lord Rayleigh's son, the present holder of the title and Professor of Physics at the Imperial College.

About the same time (*Philosophical Magazine*, 1871) Lord Rayleigh proposed a very interesting theory of double refraction. Treating the ether as an elastic solid, refraction at the interface of two media may be caused by differences either of density or of rigidity in the two, or, of course, both these differences may exist. It would be, perhaps, better to say that, owing to the action between the ether and matter, the terms brought into the equation of motion may affect both the inertia and rigidity of the ether. Lord Rayleigh, following Green and Cauchy, assumes that the rigidity is unaffected, the statical properties of the ether are the same in all media; the dynamical properties differ. The effect of the matter is solely to increase the effective or apparent density of the ether and, further,—this is the basis of his theory—in a crystal, this increase of density depends on the direction in which the ether particle concerned is vibrating. On this, combined with the principle that the vibrations are in the wave front, Lord Rayleigh develops his theory. The laws, however, to which it leads differ from experimental results to an extent which the present writer was able to show, in 1876, was in excess of the possible error of those experiments. A modification of the theory, however, leads to Fresnel's combination, which the experiments verified.

Another paper, published about the same time, "On the Reflection of Vibrations at the Confines of two Media between which the Transition is gradual" (*Proceedings of the London Mathematical Society*, Vol. XI), is of special interest just now, in view of the remarkable results announced by Mr Twyman in his letter to *Nature* of November 20th last. Reference may also be made to the paper in *Nature* for November 17th, 1891, entitled "Experiments on Colour." In this he described a simple form of colour matching apparatus and some striking differences in the colour vision of a number of observers.

In 1879, while Professor at Cambridge, Lord Rayleigh devoted considerable attention to optical instruments and, in particular, to the theory of their resolving power. The principle enunciated in one of his papers that "A double line cannot be fairly resolved unless its components subtend an angle exceeding that subtended by the wave-length of light at a distance equal to the horizontal aperture" is of fundamental importance in spectroscopic work. The paper deals also with the resolving power of a grating. The whole series of papers entitled "Lord Rayleigh's Investigations in Optics" in the *Philosophical Magazine* for 1879–80 will well repay careful study.

A later paper, *Philosophical Magazine*, August, 1881, attacks, on the Electromagnetic Theory, some of the problems treated ten years before from the elastic solid standpoint. In this theory, also, two constants occur, the one defining the magnetic, the other the electric properties of the medium—electrical phenomena

lead to the conclusion that the magnetic constant is very approximately the same for all non-magnetic media, while the electric constant changes from medium to medium and the phenomena of reflection and refraction, scattering, etc. are worked out on this assumption.

Throughout his life Lord Rayleigh's interest in Optics continued. His last paper before our Society has been mentioned. He showed that an error would probably be produced by the ship's motion; to obtain an upper limit to its amount, he discussed the error caused by the motion of an infinite circular cylinder moving at right angles to its length and showed that, at a speed of 25 miles an hour, the disturbance might amount to one-twentieth of a second. In his reply, he welcomed the suggestion made by Commander Baker, during the discussion, that practical tests which, perhaps, might be carried out in the wind tunnel at Teddington should be made.

The above is but a brief and partial account of his optical work. For nearly 50 years he was among the leaders of English Physicists. Words from the book of Wisdom "Thou hast ordered all things in measure and number and weight" form the motto of the last volume of his collected papers. Few have done more to explain to mankind the import of much of that order.

R. T. G.